CHAPTER 4

# Fuel Calculations

The steaming range capabilities of nuclear ship fuels appear in excess of 250,000 nautical miles. The *SAVANNAH*, for example, expects to achieve more than 350,000 miles, or better than 600 days of operation . . . without refueling. This she expects to do on an initial U-235 inventory of 725 pounds! The fact, however, that the fuel "when consumed" may cost as much as $7500 per pound and that all of it must be built-in at the start, suggests calculational care when trying to determine in advance the fuel requirements for a nuclear ship voyage. Due to the nature of fission, its self-poisoning and control, one cannot so readily estimate the fuel requirements as in the case of oil-fired ships. Many nuclear calculations and concepts unavoidably are involved. A review of the fundamentals behind these calculations will further an understanding of the fuel technology involved.

## 4-1 Selection of Enrichment

As we mentioned in Sec. 3-2, the degree of fuel enrichment has an important bearing on the cost of nuclear fuel. The lower the enrichment, the lower the fuel first cost, though the subsequent cost of fabrication into fuel elements remains unchanged. On the other hand, the higher the enrichment, the greater the number of new fission-sustaining neutrons produced per thermal neutron absorbed in the fuel. The larger this number, called "eta" (Greek letter $\eta$), the smaller is the size of reactor necessary. Consequently, the selection of a fuel enrichment level invokes a compromise between direct fuel cost and the reactor design.

Eta is an important property in determining the fission multiplication of a fuel and, as such, it is a preliminary index of fuel efficiency. For any degree of enrichment (%E) of U-235, eta can be determined from the following expression:

[Eq. 4-1]*

$$\eta = \frac{1400\,(\%E)}{675\,(\%E) + 2.75\,(100 - \%E)}$$

* Ref: *Nuclear Reactor Physics*, R. L. Murray, Prentice-Hall, 1957, p. 99.

Computed values of eta for various enrichments are listed in Table 4-1. Note that at 5% enrichment, for example, we have an eta-value better than 90% that of fully enriched U-235. As a matter of interest, we should be aware that the *SAVANNAH* uses close to 5% U-235 fuel.

We should be careful to note that eta is not the true number of neutrons produced in each act of fission; it is the *net* number produced per neutron absorbed in the fuel. Actually, each U-235 fission produces 2.48 neutrons, but U-235 fission occurs only 85% of the time. That is, about 15% of the neutrons absorbed in U-235 produce U-236 . . . which is non-fissionable. Hence, eta is a fissioning efficiency factor. It is a fundamental term that appears in all nuclear fuel calculations.*

**Table 4-1. Eta-Values of Nuclear Ship Fuels**

| %E | $\eta$ |
|---|---|
| NAT | 1.32 |
| 1 | 1.48 |
| 2 | 1.73 |
| 3 | 1.84 |
| 4 | 1.89 |
| 5 | 1.93 |
| 10 | 2.01 |
| 25 | 2.05 |
| 50 | 2.07 |
| 75 | 2.075 |
| FULL | 2.08 |

## 4-2 Fission Multiplication Factors

The over-all property of a nuclear fuel—and its neutron moderator—which provides the basis for a self-sustaining fission reaction is the fission multiplication factor, called "k." When k is exactly equal to one, we have a self-sustaining fuel system, called "criticality." In any practical reactor, with a k of exactly one, the reactor would operate for a short time, then would shut itself down. This would happen because, as the criticality fuel is burned, fission product poisons are built up. Soon, the newly created poisons absorb more neutrons than the fuel, and the reactor goes subcritical. To avoid this, an excess k is always sought.

To achieve an excess k, we build it up mathematically from two other forms of k. The basic building-block k is called "k-infinity" ($k_\infty$). This infinite multiplication factor presumes a fission multiplying region of infinite size. Such a region is not concerned with the loss of neutrons by leakage. In a practical reactor, of course, neutrons inevitably leak from the fission multiplying region and, hence, we speak of a finite k or "k-effective" ($k_e$). After we have taken neutron leakages into account, we have to compensate for the burnup of fissionable nuclei, and we have to provide for the override of fission product poisons, temperature effects, and control tolerances. When we do all of this, we arrive at a k-excess, called "delta-$k_e$" ($\Delta k_e$). For the time being, we shall ignore $k_e$ and $\Delta k_e$, and consider only $k_\infty$.

* Other fissionable fuel eta-values are: $U^{233} = 2.31$, $Pu^{239} = 2.03$.

### 4-3 K-infinity for Thermal Fission

The relationship which takes into account all factors in computing $k_\infty$ for thermal fission is

[Eq. 4-2]

$$k_\infty = \eta \varepsilon f p$$

In this expression, $\eta$ (eta) already has been discussed. The term $\varepsilon$ (epsilon) is called the "fast fission factor." The terms "f" and "p" will be discussed later.

Epsilon takes into account the fact that a new (fast) neutron born in U-235 fission is capable of fissioning a U-238 atom before it (the new neutron) is slowed down to thermal energies. We said previously (Sec. 1-1) that U-238 generally is non-fissionable. But to be precisely correct, U-238 is fissionable, though *only* by fast neutrons: those greater than 1 Mev.* The probability that this will happen is $\varepsilon - 1$. A typical value of epsilon is 1.03 which, in other words $(\varepsilon - 1)$, is a 3% fast fission effect.†

Each U-238 atom occasionally fissioned by a fast neutron also produces new neutrons. These U-238 neutrons combine with those from the thermal fission of U-235. Hence, the product eta-epsilon represents the ability of a selected nuclear fuel to reproduce new neutrons. Using, for example, a 5% $\eta$-value from Table 4-1 (i.e., 1.93) and the typical $\varepsilon$-value of 1.03, the neutron reproduction ability of the fuel would be 1.99, or for simplicity, say, 2. That is, every neutron that is absorbed in the fuel produces a *net number* of two new fast neutrons to continue the fission cycle. Thus, now,

[Eq. 4-3]

$$k_\infty = 2fp$$

where f and p can now be explained.

The term f is called the "thermal utilization factor" of a fission multiplying region. That is, f is the fractional absorption of the eta-epsilon neutrons in the fuel, after taking into account the neutron absorption losses in the moderator and in all other materials in the reactor core. These losses are generally computed when the neutrons have been thermalized. The fraction f, therefore, is one index of the neutron competition in the core.

A second index of this competition is p, called "resonance escape probability." While the neutrons are slowing down from fission to thermal energies, they encounter so-called resonance peaks in the U-238 portion of the fuel. These resonance peaks capture the neutrons and prevent them from fissioning the U-235. We attempt to avoid any resonance captures by proper fuel-moderator design. The probability of our success is p.

* The U-238 fast fission probability is 0.3 barns compared with 580 barns for the thermal fission of U-235. So, we are dealing essentially only with thermal fission events.

† Ref: *Introduction to Nuclear Engineering,* R. Stephenson, McGraw-Hill, 1954, pp. 112-113.

From Eq. 4-3 it is desirable that f and p be as large as possible, in order to maximize $k_\infty$. But, as we shall see, the fuel design factors that increase f decrease p . . . and vice versa.

## 4-4 The Fuel "Unit Cell"

Before we are able to determine the values of f and p, it is necessary that we know something about the fuel element geometry, the moderator composition, and the relative atomic and volume proportions of each. It is also necessary to know the relative proportion of poisons that the neutrons will encounter. The term "poison" used here is a lumped composite which includes the non-productive effects of neutron absorption by coolant, structural materials, control rods, and fission residues. From the nuclear standpoint, everything in the reactor except fuel and moderator is a neutron poison.

One of the best physical approaches to determining f and p is to consider a "unit cell" of fuel, moderator, and poison. The unit cell—centered about the fuel itself—is one of many, many such cells which comprise the total core of a reactor. This concept permits certain simplifications from a mathematical standpoint. For ship-type reactors, where the moderator and coolant are one and the same (water), a unit cell would appear as shown in Fig. 4-1.

From Fig. 4-1, and from various mathematical rearrangements based on the definition of thermal utilization, an expression for f may be written as

[Eq. 4-4]*

$$f = \frac{\sigma_F}{\sigma_F + \phi_M/\phi_F \, N_M/N_F \, V_M/V_F \, \sigma_M + \phi_P/\phi_F \, N_P/N_F \, V_P/V_F \, \sigma_P}$$

where

$\sigma$ = (small sigma) neutron absorption per atom

$\phi$ = (phi) average thermal neutron flux

N = number of atoms per cubic centimeter

V = fraction volume of unit cell

and subscripts F, M, P = fuel, moderator, and poison, respectively.

In Eq. 4-4, we note that the only absolute values needed are $\sigma_F$, $\sigma_M$, and $\sigma_P$. These can be obtained from nuclear engineering handbooks for each reactor material.† The other information needed: average flux ratios ($\phi_M/\phi_F$; $\phi_P/\phi_F$), atom ratios ($N_M/N_F$; $N_P/N_F$), and volume ratios $V_M/V_F$; $V_P/V_F$). Obtaining these ratios requires extensive calculations, empiricisms, experimentation, and unit cell layouts with various fuel element sizes. The net result, however, is that typical values of f may vary from 0.60 to 0.90.

* Ref: *Nuclear Reactor Physics*, R. L. Murray, Prentice-Hall, 1957, pp. 84 ff.

† For example: *Nuclear Engineering Handbook*, H. Etherington, McGraw-Hill, 1958.

In the case of determining p (resonance escape probability), the expression for a unit cell is

[Eq. 4-5]*

$$p = e^{-\phi_F/\phi_M \, N_F/N_M \, V_F/V_M \frac{\sigma_r}{\xi \, \sigma_s}}$$

where

$e = 2.716$
$\sigma_r$ = resonance absorption in U-238
$\sigma_s$ = scattering "absorption" in moderator
$\xi$ = (xi) average logarithmic neutron energy change per moderating event

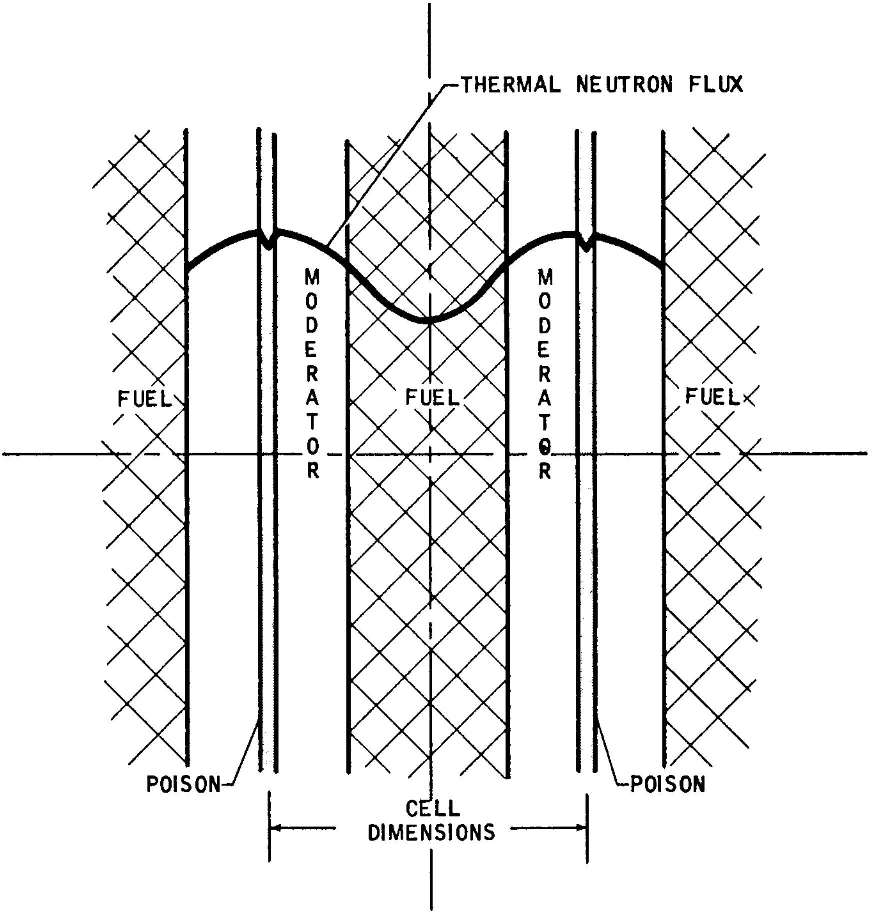


Fig. 4-1 Representative "Unit Cell" for a Ship Reactor

* Ref: *Nuclear Reactor Physics,* R. L. Murray, Prentice-Hall, 1957, pp. 92 ff.

Here, again, the flux ratios, atom ratios, and volume ratios appear, but this time in *reciprocal form* to those in Eq. 4-4. The values for $\sigma_s$ and $\xi$ can be obtained from nuclear tables, but $\sigma_r$ has to be calculated for each type of unit cell. In the end, typical values of p may also vary from 0.60 to 0.90.

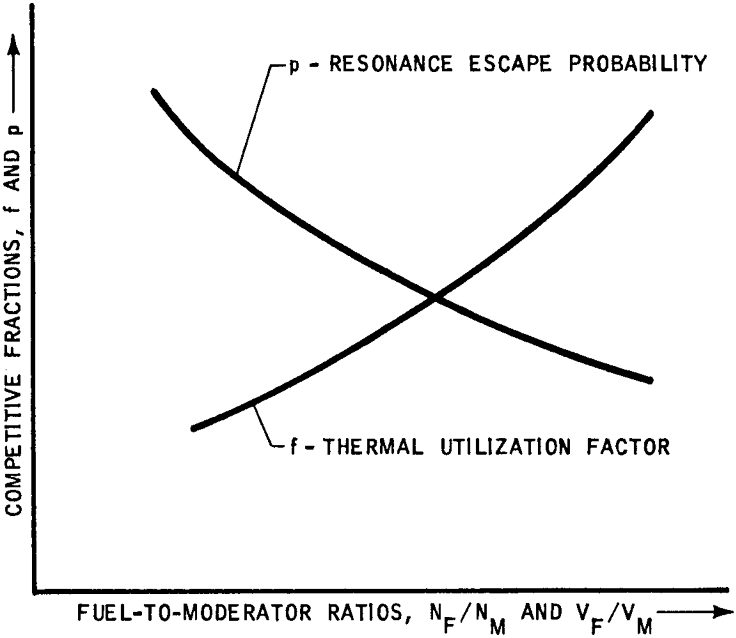


Fig. 4-2 Contradictory Influences of Fuel and Moderator on Fission Multiplication

Although f and p vary within the same fractional range, each varies in a direction contrary to the other. This is evident in Eqs. 4-4 and 4-5. In the former equation, the ratios $N_M/N_F$ and $V_M/V_F$ appear, whereas in the latter equation the ratios are reversed: $N_F/N_M$ and $V_F/V_M$. This reciprocity tells us that the proportions of fuel and moderator in a reactor have contradictory effects on f and p. That is, the greater the proportion of fuel relative to the moderator, the larger is f . . . but the smaller is p (see Fig. 4-2). Yet, we must use both f and p when determining $k_\infty$ (Eq. 4-3). Consequently, in order to make $k_\infty$ as large as possible, we seek the largest product of f and p. To do this, we have to perform a whole series of unit cell calculations.

## 4-5 Finite Neutron Leakages

K-infinity, it should be remembered, is a measure of the fission multiplication in an infinite region of unit cells. But we are really interested only in finite regions, such as are found in practical reactors. In such regions, the fission-produced neutrons undergo two forms of leakage. One, they leak from the reactor while slowing down from fast energies, called "fast leakage" (recall Fig. 1-3). And, two, they leak from the reactor while scurrying around at thermal energies in search for fuel nuclei, called "thermal leakage." Those neutrons which do not leak from the system are the useful ones and, hence, the term "non-leakage probability," symbolized by $\mathcal{L}$. The fast non-leaking probability is $\mathcal{L}_f$, while the thermal non-leakage probability is $\mathcal{L}_t$. With these two terms in mind, our k of interest (k-effective) becomes

[Eq. 4-6]

$$k_e = k_\infty \mathcal{L}_f \mathcal{L}_t$$

We now bring forth two famous theories in nuclear physics which possess remarkable simplicity for approximating the complex phenomena of neutron motion. One, the Fermi-Age theory, gives $\mathcal{L}_f$ for a bare (unreflected) reactor as

[Eq. 4-7]

$$\mathcal{L}_f = e^{-B^2\tau}$$

where

$B^2$ = a "buckling" factor dependent upon the reactor geometry*

$\tau$ = (tau) Fermi "age" of neutrons, a measure of the distance that neutrons travel from fission energy to thermal energy

The second approximation, Diffusion theory, gives $\mathcal{L}_t$ for a bare reactor as

[Eq. 4-8]

$$\mathcal{L}_t = \frac{1}{1 + B^2L^2}$$

where $L^2$ = composite thermal diffusion length of neutrons in the moderating, fuel, and poison regions.

Combining Eqs. 4-7 and 4-8 into the form of Eq. 4-6, we have by Age-Diffusion theory an expression for k-effective as

[Eq. 4-9]

$$k_e = \frac{k_\infty e^{-B^2\tau}}{1+B^2L^2}$$

$$= 1 \text{ for criticality}$$

The effect of both leakage terms is to reduce $k_\infty$. So, unless $k_\infty$ is greater than 1, we would not get a very useful reactor.

Numerical values for tau and $L^2$ are tabulated in nuclear handbooks for various moderating materials. These tabulated values, however, have

* The term "buckling" arises from the fact that $B^2$ is a measure of the bending (or buckling) of the neutron flux at any point in a reactor.

to be corrected for neutron absorption in the fuel and poisons. Typical corrected (round figure) values for water systems might be $\tau = 50\ cm^2$; $L^2 = 5\ cm^2$. With $k_\infty$, $\tau$, and $L^2$ known, the only unknown in Eq. 4-9 is $B^2$: the buckling factor.

## 4-6 Minimum Fuel for Criticality

Eq. 4-9 is the simplest relationship available for approximating the minimum fuel requirements for criticality in a reactor. This equation relates the physical characteristics of the fission multiplying region (fuel, moderator, and poison)—as given by $k_\infty$, $\tau$, and $L^2$—to the physical size and shape of the reactor, which is identified by $B^2$. But, unfortunately, the determination of $B^2$ must be done by trial and error. The usual approach is to start with the approximation that

[Eq. 4-10]*

$$B^2 \approx \frac{\ln k_\infty}{L^2 + \tau}$$

and then work back and forth until $B^2$ converges on itself to satisfy Eq. 4-9 (i.e., when $k_e = 1$). This trial and error can be done by hand computations, though it is more convenient to do so with machine computers.

But when $B^2$ is found, it is a meaningless number, until we have decided on the over-all geometry (shape) of the reactor core. For ship reactors, the usual shape is a right cylinder of radius R and height H. Now, the expression which relates $B^2$, R, and H is

[Eq. 4-11]†

$$B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{3.142}{H}\right)^2$$

Using the *SAVANNAH* core as an example, H is approximately 2.5 R.‡ Substituting this into Eq. 4-11 and rearranging, we get

[Eq. 4-12]

$$R^2 = \frac{7.37}{B^2}$$

So, once we have $B^2$, we can determine the criticality radius of our cylindrical reactor core, and from this radius (when H = 2.5 R) we can determine the criticality volume.

* The "$\approx$" symbol means "approximately equal to."

† Ref: *Principles of Nuclear Reactor Engineering*, S. Glasstone, Van Nostrand, 1955, Table 3.7.

‡ Ref: Paper No. 96, "Reactor Physics and Core Design of the Merchant Ship Reactor," Wood and Levine, American Nuclear Society Conference, Chicago, March, 1958.

To determine the criticality fuel—often called "critical mass"—we need to know the volume fraction of the unit cell (Fig. 4-1) which is fuel. This fuel fraction is influenced largely by non-nuclear considerations such as heat transfer, fuel cladding, structural materials, and other features of core design. Nevertheless, once the volumetric fuel fraction is known, the critical fuel mass can be determined directly from the criticality volume of the core.* In general, the critical fuel mass will decrease for increasing core size up to some minimum value, then will increase rapidly thereafter (see Fig. 4-3).

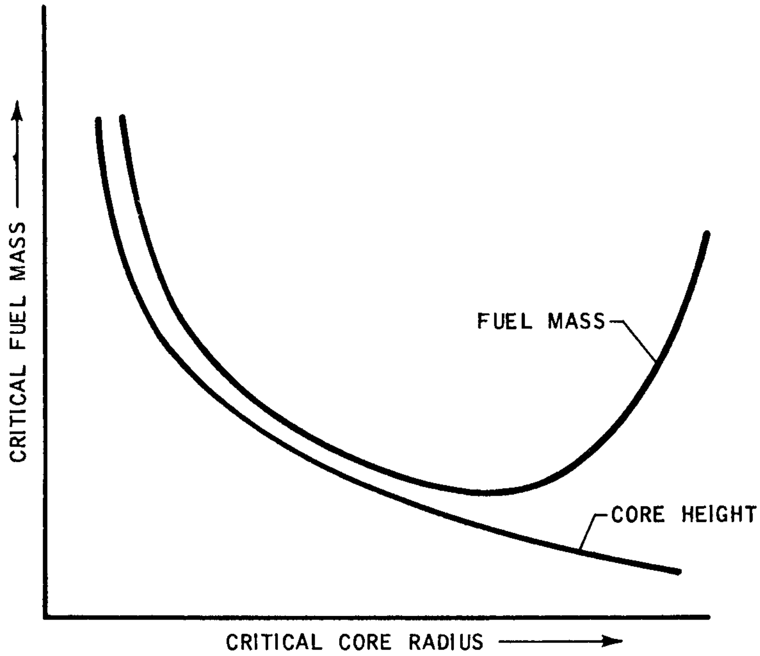


Fig. 4-3 Generalized Criticality Fuel Conditions for a Cylindrical Core

Actually, the Age-Diffusion equation (Eq. 4-9) is accurate only to about 80 or 90% for calculating critical size and critical mass.† The Age-Diffusion theory presumes monoenergetic (all one energy) fission neutrons which slow down continuously from fission to thermal energies. As one can readily imagine, the neutrons born in fission are not all of one energy (see Fig. 4-4) and they do not slow down continuously. Nevertheless,

* In the case of the *SAVANNAH*, the core volume fraction of fuel (U-235 plus U-238) is 0.2455. Ref: "The Power Plant for the First Nuclear Merchant Ship," J. W. Landis, Bulletin AER-54, 1958 Nuclear Merchant Ship Symposium, Washington, D. C.

† Ref: *Introduction to Nuclear Engineering*, R. Stephenson, McGraw-Hill, 1954, p. 156.

by making theoretical simplifications, we are able to at least approximate the physical happenings in a nuclear reaction without becoming overburdened initially with mathematical detail.

Since the criticality conditions must be predicted to minute fractions of a per cent, it is necessary to resort to more complex equations than the Age-Diffusion theory. These equations are developed from so-called

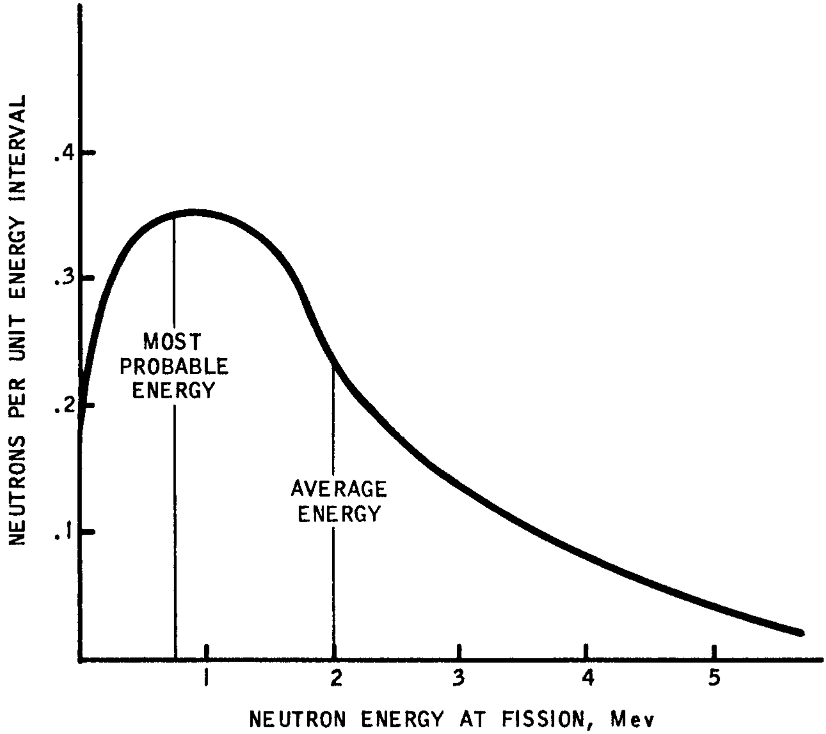


Fig. 4-4 Energy Distribution of Neutrons at Fission "Birth"

"multigroup-multiregion" theories. Each of these theories takes more and more physics phenomena into account; divides the neutron energies into smaller and smaller intervals (groups); divides the core into smaller and smaller segments (regions) . . . and so on. A typical such equation is as follows, but no attempt is made to explain its terms:*

$$\chi^i S(r) + \Sigma_R^{i-1}(r)\phi^{i-1}(r) = \phi^i(r)\,[D^i(r)\,B^{i\,2}(r) + \Sigma_a^i(r) + \Sigma_R^i(r) + t^i \Sigma_P^k] - \nabla\,[D^i(r)\,\nabla\,\phi^i(r)]$$

The net consequence is that real criticality calculations cannot be done by hand: the use of high-speed electronic computers is required.

* Ref: "WANDA: A One-Dimensional Few Group Diffusion Equation Code for the IBM-704," WAPD-TM-28, Westinghouse Atomic Power Division.

Even when machine answers are obtained, it is still necessary to verify the calculational work by criticality **experiments!** No one would dare to risk his professional reputation by guaranteeing the critical mass of a *completely new* reactor on theoretical calculations alone. Thus, the cruciality of criticality fuel for the first reactor of its class.

### 4-7 Fuel in Excess of Criticality

Up to this point, we have discussed the minimum fuel requirements to get a nuclear chain reaction going. In the case of the *SAVANNAH*, this minimum fuel (critical mass) is about 280 kg (620 pounds) U-235.* But nothing has been said, so far, about the fuel required to sustain a specified power level, nor has there been any discussion of the additional fuel to override poison buildups, temperature effects, and control tolerances. These factors necessitate fuel in excess of criticality.

Theoretically, once criticality is achieved, we could attain any power level desired (recall Fig. 1-5). But unless new fuel is added, the power rise would be of momentary duration. The fuel consumed would detract from the critical mass, and the nuclear reaction would go sub-critical. Obviously, therefore, it is necessary to provide adequate fuel to sustain the desired power level. A handy rule of thumb in this regard is the approximation that one gram of fuel fissioned produces one megawatt of heat for one day. Take the *SAVANNAH*, for example, which has a 70 MW reactor . . . with at least a 600-day core life.† At the consumption rate of 1 gm/MW-day, approximately 42 kg of "power fuel" would be required. This represents about 15% of the *SAVANNAH's* initial criticality fuel. However, better than half of the power fuel is obtained from the internal conversion of non-fissionable U-238 to fissionable Pu-239.

Every nuclear ship reactor that goes into operation builds up neutron poisons in proportion to the power level (neutron flux) at which it operates. These poisons consist largely of xenon-135, though other fission residues contribute somewhat. The poison buildup climbs to a maximum of about 5% of the criticality fuel during operation, and to a maximum of 250% after shutdown! (See Table 4-2.) Consequently, depending on the flux level of the reactor, extra fuel is required to override these poisons. The *SAVANNAH*, for example, provides about 3% fuel for poison override. This corresponds to a flux level of $10^{13}$ in Table 4-2, and assumes that the *SAVANNAH's* reactor would never be completely shut down during its active core life. (300 K)

For a clean-cold reactor, the criticality calculations are based on an ambient temperature of approximately 70°F. But when a reactor heats up to its normal operating temperature, the nuclear and density properties of its fuel, moderator, and other materials undergo change. In the case

* Kg = kilogram; 1 kg = 2.2 pounds.

† Ref: Paper No. 96, "Reactor Physics and Core Design of the Merchant Ship Reactor," Wood and Levine, American Nuclear Society Conference, March, 1958, Chicago.

of the fuel, for example, if the reactor operated at, say, 570°F, the fission probability (due to the speeded-up neutrons) would decrease. This is typified in Fig. 4-5. At the same time, the thermal expansion of the

**Table 4-2. Maximum Poisoning Factors Due to Xenon Buildup**

| Thermal Flux | During Operation | After Shutdown |
|---|---|---|
| $10^{12}$ | 0.007 | 0.007 |
| $10^{13}$ | 0.030 | 0.066 |
| $10^{14}$ | 0.046 | 0.275 |
| $10^{15}$ | 0.048 | 2.500 |

fuel would result in a fewer number of fissionable nuclei per unit fuel volume. Both of these effects reduce the $k_\infty$ fission multiplication of the fuel.

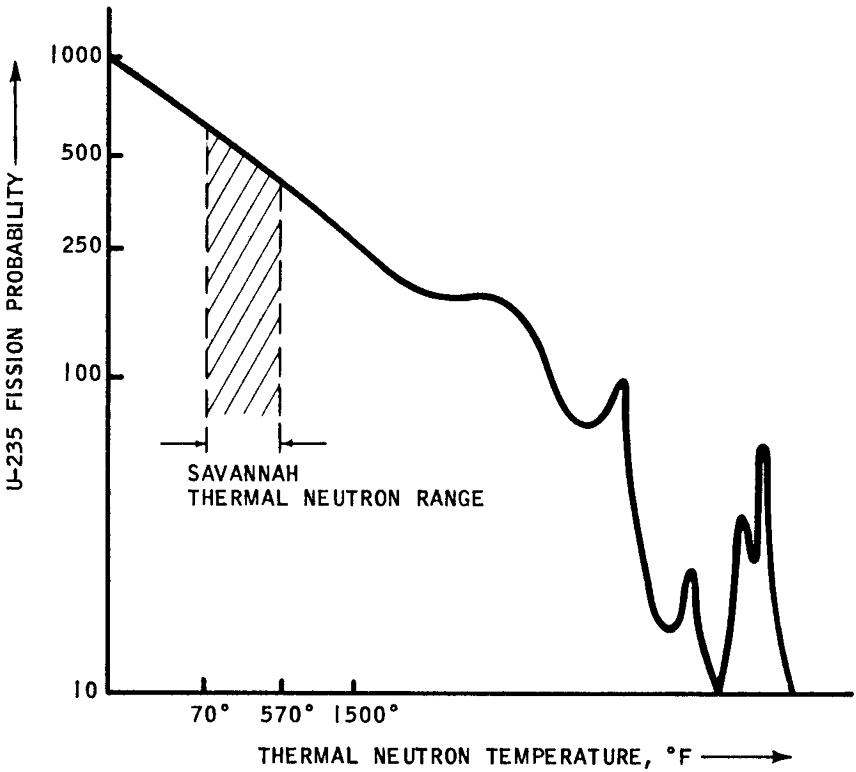


Fig. 4-5 Decreasing Fission Probability with Increasing Neutron Temperature

Temperature conditions also alter the nuclear properties and densities of the moderator, structure, and poison materials. In particular, $L^2$ and $B^2$ are changed in the criticality equations. The over-all temperature influence on the criticality fuel can be approximated from the relation

[Eq. 4-13]*

$$\Delta C = -\frac{B^2L^2}{2\,k_\infty T} - 4\alpha\,\frac{(k_\infty - 1)}{k_\infty}$$

where

$\Delta C$ = fractional change in criticality fuel per °F
T = absolute temperature °R (460 + °F)
$\alpha$ = (alpha) mean weighted coefficient of expansion of fuel, moderator, and structure

and

$B^2$, $L^2$, and $k_\infty$ are based on the operating temperature

For water reactors of the *SAVANNAH* type, the $\Delta C$ of Eq. 4-13 approximates $-20 \times 10^{-5}$ per °F. In other words, to compensate for a 500-degree temperature rise from ambient startup, the temperature fuel required would be about 10%.

To take care of off-design tolerances in the manufacture of fuel elements, control rods, and coolant flow areas, a slight excess fuel is added. This tolerance fuel may approximate 2% of the criticality fuel, depending on the design philosophies used.

Thus, we see that in addition to the criticality fuel we must provide an excess amount of fuel for the purpose of:

- sustaining power
- overriding poison buildups
- compensating for temperature effects
- allowing for manufacturing tolerances

For nuclear ships, the total excess fuel could approach 25% of the criticality fuel. The *SAVANNAH*, for example, has an excess fuel of 18.5%. This brings her total initial fissionable inventory of U-235 to about 330 kg (725 lb).†

## 4-8 Control of Excess Fuel

All of the criticality and excess fuel is built into a reactor as part of its original design. But if all of this fuel were built in-without any form of controlling it, we would have a difficult situation on our hands. In other

* Ref: *Principles of Nuclear Reactor Engineering*, S. Glasstone, Van Nostrand, 1955, pp. 259 ff.

† When specifying nuclear ship fuel requirements, one should be careful to distinguish between the U-235 and U-238 contents. Both are included in the initial loading of "fuel."

words, k-effective ($k_e$) would be considerably greater than one: a condition of supercriticality. Thus, we would breed neutrons far in excess of immediate needs, and the reactor would run away.

If we generalize on the neutron multiplication in a reactor, the time rate of change of neutrons per unit cell volume is represented by

[Eq. 4-14]

$$\frac{dn}{dt} = \text{production—leakage—absorption}$$

Eq. 4-14 means that we can control the excess productivity of neutrons in two ways: (1) increasing the neutron leakage, (2) increasing the (non-fission) absorption of neutrons. In the former, we could cut holes in the reactor, valve these holes, and vary the neutron leakage. This control method is impractical . . . and hazardous. Thus, we resort to the second method, that of control by neutron absorption. This is usually done with poison-type control rods (previously mentioned).

Now, recall Sec. 4-2 in which the excess fission multiplication was denoted by $\Delta k_e$ (delta k-effective). By definition, this $\Delta k_e = k_e - 1$. If $\Delta k_e$ is small enough, the neutron response is

[Eq. 4-15]

$$n = n_o e^{(\Delta k_e/l)t}$$

where

$n$ = neutron density at any time t
$n_o$ = neutron density at time zero
$l$ = average neutron lifetime from fission to fission
$t$ = time in seconds

Note the similarity between Eq. 4-15 and Eq. 1-3 in Sec. 1-10: the n and $\phi$ are comparable, but not equivalent.* The $T^*$ (reactor period) is $l/\Delta k_e$, where $l$ is essentially constant. In explaining Eq. 1-3, we stated that ". . . The increase or decrease in $T^*$ is established by the number of control rods withdrawn or inserted." Now, we see that what we are actually controlling is $\Delta k_e$. When we have the capability of doing this with precision, we can make $\Delta k_e$ smaller and smaller, thereby always restoring $k_e$ to 1: criticality. We do this by assigning to each control rod a specific value of $\Delta k_e$, usually around 0.7%. When the sum of all the control rod $\Delta k_e$'s is equivalent to the total excess fuel built into the reactor, we have a controllably safe situation.

## SUMMARY

We have seen that one of the principal selection features in the type of nuclear fuel used is its enrichment level. The nuclear importance lies in the number of net neutrons produced for each neutron absorbed in the fuel (called eta). The

* n (neutron density) is $n/cm^3$, where $\phi$ (neutron flux) is $n/cm^2$-sec.

higher the enrichment, the higher the eta, though to an ever diminishing extent. A 5% enrichment, for example, gives a U-235 eta-value of better than 90% that of fully enriched fuel. Hence, fuels containing up to 5% U-235 represent the practical range of enrichment interest for nuclear merchant ships.

We have also seen that in order to determine criticality fuel requirements, we first have to determine a fission multiplication factor called k-infinity. This factor necessitates the simplification of a fuel "unit cell" in which the atomic proportions of fuel, moderator, and poison are estimated, and for which the lattice geometry and materials are selected. With this information—usually obtained by trial and error and by reference to similar prior designs—we compute the U-235 thermal fission utilization factor f and the U-238 resonance escape probability p. The core design features that increase one decrease the other. The resulting k-infinity, however, applies to an infinite fission multiplying region without leakages of any kind.

When neutron leakages are taken into account, the multiplication factor becomes k-effective which is of more practical interest. To attain fuel criticality, $k_e$ must equal one (i.e., unity). Two types of neutron leakage are involved; one, while the neutrons are slowing down from fast energies to thermal; and the other, while the neutrons are scurrying around thermally in search for fissionable fuel nuclei. Two famous theories permit remarkable simplifications in estimating these leakages, but certain inaccuracies are involved. Improved calculational accuracy is attained by multigroup-multiregion theories which take into account many details of neutron behavior. With all of the analytical sophistication that the human mind can muster, it is still necessary that the final criticality fuel (for the first reactor of its class) be confirmed by experiment.

Once the criticality fuel is determined, unless fuel-in-excess is supplied, the nuclear reaction would die out on its own. Four factors account for this, namely: (1) portions of the critical mass are depleted; (2) fission residue poisons build up; (3) operating temperatures change fissioning and moderation characteristics; and (4) manufacturing tolerances can throw off criticality predictions. To offset these factors, a total fuel in excess of criticality approximating 25% may be required. In the case of the *SAVANNAH*, for example, 18.5% excess fuel is supplied.

Excess fuel, however, means a runaway reactor unless some method of control is provided. This control is provided by neutron absorption (control) rods which divide up the total excess fuel into small delta-$k_e$ ($\Delta k_e$) components. We require that all control rods act together to maintain the reactor safely critical.